Sequences

(ch. 10.1)

Wednesday, April 5, 2023 8:56 AM

sequence : $\{a_n\}$, also $a_1, a_2, a_3, a_4, \ldots$, is an ordered list of real #'s ($a_i \in \mathbb{R}$) indexed by natural #'s

(based on) $n = 1, 2, 3, 4 \ldots$ (always $\mathbb{R}$)

ex) $a_1 = 0.01$, $a_2 = 0.012$, $a_3 = 0.0122$, $a_4 = \ldots$, keep listing

* not always feasible way to create sequence *

sequences defined by closed formula:

ex 1) $a_n = 1/n$

$a_1 \to n = 1$, a_2, a_3, a_4, a_5, a_6, a_7

$a_1 = 1/1 = 1$, $1/2$, $1/3$, $1/4$, $1/5$, $1/6$, $1/7 \ldots 1/\infty \to \boxed{0}$

1, 0.5, $0.\overline{3}$, 0.25, 0.2

* constanly ↓ / getting closer & closer to zero *

ex 2) $a_n = \left(1 + \frac{1}{n}\right)^n$

a_1, a_2, a_3, a_4, a_5

2, $9/4$, $64/27$, $(1 + 1/4)^4$, $(1 + 1/5)^5 \ldots$

2, 2.25, $2.37 \longrightarrow 2.\text{something} \longrightarrow \boxed{e}$ (≈ 2.718)

ex 3) $a_n = \sqrt{n}$

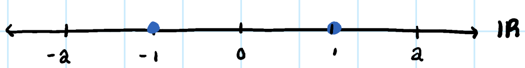
$\sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \ldots, \sqrt{9}, \ldots, \sqrt{16} \to \infty$

$1, 1.4, 1.7, 2, \ldots, 3, \ldots, 4$ $\boxed{\text{DNE}}$

* no limit / if going to ∞ or $-\infty$ / not bounded *

ex 4) $a_n = (-1)^n = -1, 1, -1, 1$

$a_n = \begin{cases} 1 & \text{if } n = \text{even} \\ -1 & \text{if } n = \text{odd} \end{cases} = \boxed{\text{DNE}}$



* oscillating : going between 2 points / 2 different limits / no limit for a_n *

ex 5) $a_n = \cos(\pi(n-1))$

(same as above)

$\cos(\pi(1-1))$
$\cos(0) = 1$

$\cos(\pi(2-1))$
$\cos(\pi) = -1$

$\boxed{\text{sequence } (a_n) \text{ is given}}$

converges (has lim) | diverges (lim = DNE)

- reasons : (thm. 1, 2, 3, 4, 5, 6 in book)
  - sandwich theorem
  - MCT (monotone convergence theorem)

- reasons :
  - unbounded ($-\infty$ or ∞)
  - oscillating (converges to 2 different limits)

- what is lim ?

steps :

1) try a few #'s / terms

2) does it converge or diverge ?

3) determine lim if possible

* can argue a_n converges without stating lim *